ELEG5481 Signal Processing Optimization Techniques
Assignment 1

Feb. 1, 2013

Deadline: Feb. 20, 2013

Q1. (20%) Show that following functions are norms.
(a) $f(x) = \|Tx\|$, where$\|\cdot\|$ is a norm, $T \in \mathbf{R}^{n\times n}$ is nonsingular.
(b) $f(X) = \max_{1\le i\le m}\{\sum_{j=1}^{n} |X_{ij}|\}$, where $X \in \mathbf{R}^{m\times n}$.
(c) $f(X) = \sup\{\|Xu\|_a \mid \|u\|_b \le 1\}$, where $\|\cdot\|_a$ and $\|\cdot\|_b$ are norms, and $X \in \mathbf{R}^{m\times n}$.

Q2. (20%) Prove that

$$\frac{1}{\sqrt{n}}\|x\|_1 \le \|x\|_2 \le \|x\|_1.$$

Q3. (20%) The matrix p-norm is defined as

$$\|X\|_p = \max\{\|Xu\|_p \mid \|u\|_p \le 1\}.$$

Show the following results
(a) $\|X\|_\infty = \max_{1\le i\le m}\{\sum_{j=1}^{n} |X_{ij}|\}$.
(b) $\|X\|_2 = \sigma_{\max}(X) = \sqrt{\lambda_{\max}(X^T X)}$, where $\sigma_{\max}(X)$ denotes the largest singular value of X, and $\lambda_{\max}(A)$ the largest eigenvalue of A.

Q4. (20%) Verify the following: Given that $A \in \mathbf{C}^{n\times n}$ is Hermitian,
(a) $\|A\|_F^2 = \sum_{i=1}^{n} \lambda_i^2$.
(b) $\det(A) = \prod_{i=1}^{n} \lambda_i$
(c) $\mathrm{tr}(A) = \sum_{i=1}^{n} \lambda_i$.

Q5. (10%) Show the following results.
(a) Suppose $A \in \mathbf{R}^{n\times n}$ is positive semidefinite. If $A_{i,i} = 0$ for some i, then $A_{j,i}$ and $A_{i,j}$ are zeros for all $j = 1, \ldots, n$.
(b) Suppose $A \in \mathbf{C}^{n\times n}$ is positive semidefinite. If $A_{i,i} = 1$ for $i = 1, \ldots, n$, then $|A_{i,j}| \le 1$ for any i, j.

Q6. (10%) Let $X \in \mathbf{S}^n$. Show that

$$\mathrm{tr}(XY) \ge 0 \quad \forall\, Y \in \mathbf{S}_+^n$$

if and only if X is PSD.