ELEG5481 Signal Processing Optimization Techniques
Tutorial 9

Mar. 31, 2013

Q1. (a) Suppose that we are given a set of discrete points $\{x_i\}_{i=1}^{K+L}$. Show that there exists an hyperplane $a^T x = b$ that strictly separates $\{x_i\}_{i=1}^{K}$ from $\{x_i\}_{i=K+1}^{K+L}$, i.e.,

$$a^T x_i < b, \quad i = 1, \ldots, K, \qquad a^T x_i > b, \quad i = K+1, \ldots, K+L,$$

if and only if there exist two parallel hyperplanes $\tilde{a}^T x = \tilde{b} - 1$ and $\tilde{a}^T x = \tilde{b} + 1$ which separate $\{x_i\}_{i=1}^{K}$ from $\{x_i\}_{i=K+1}^{K+L}$ in the sense that

$$\tilde{a}^T x_i \leq \tilde{b} - 1, \quad i = 1, \ldots, K, \qquad \tilde{a}^T x_i \geq \tilde{b} + 1 \quad i = K+1, \ldots, K+L.$$

(b) Suppose that we are given $K + L$ ellipsoids

$$\epsilon_i = \{P_i u + q_i \ \mid \ \|u\|_2 \leq 1\}, \quad i = 1, \ldots, K+L,$$

where $P_i \in \mathbf{S}^n$. We are interested in finding a hyperplane that strictly separates $\epsilon_1, \ldots, \epsilon_K$ from $\epsilon_{K+1}, \ldots, \epsilon_{K+L}$, i.e. we want to compute $a \in \mathbf{R}^n$, $b \in \mathbf{R}$ such that

$$a^T x + b < 0 \text{ for } x \in \epsilon_1 \cup \ldots \cup \epsilon_K, \quad a^T x + b > 0 \text{ for } x \in \epsilon_{K+1} \cup \ldots \cup \epsilon_{K+L},$$

or prove that no such hyperplane exists. Express this problem as an SOCP feasibility problem.

Q2. Show how to convert the conic form SDP to the standard form SDP.
Conic form:

$$\begin{aligned} \min_{x \in \mathbf{R}^n} \quad & c^T x \\ \text{s.t.} \quad & x_1 F_1 + \ldots + x_n F_n + G \preceq 0, \\ & Ax = b, \end{aligned}$$

where $G, F_1, \ldots, F_n \in \mathbf{S}^k$, and $A \in \mathbf{R}^{m \times n}$.
Standard form:

$$\begin{aligned} \min_{X \in \mathbf{S}^n} \quad & \mathbf{tr}(CX) \\ \text{s.t.} \quad & \mathbf{tr}(A_i X) = b_i, \ i = 1, \ldots, p \\ & X \succeq 0, \end{aligned}$$

where C, $A_1, \ldots, A_p \in \mathbf{S}^n$.