ELEG5481 Signal Processing Optimization Techniques
Tutorial 8

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Q1. Show that $X = B^T A^{-1} B$ solves the SDP

$$\begin{aligned} \min_{X \in \mathbf{S}^n} \quad & \mathbf{tr} X \\ \text{s.t.} \quad & \begin{bmatrix} A & B \\ B^T & X \end{bmatrix} \succeq 0, \end{aligned}$$

where $A \in \mathbf{S}^m_{++}$ and $B \in \mathbf{B}^{m \times n}$ are given data. Conclude that $\mathbf{tr}(B^T A^{-1} B)$ is a convex function of (A, B), in the domain of $\mathbf{S}^n_+ \times \mathbf{R}^{m \times n}$.

Q2. Formulate the following complex-valued infinity norm minimization problem as an SOCP.

$$\begin{aligned} \min_{x \in \mathbf{C}^n} \quad & \|x\|_\infty \\ \text{s.t.} \quad & Ax = b, \end{aligned}$$

where $A \in \mathbf{C}^{m \times n}$ and $b \in \mathbf{C}^m$. Hint: transform the complex-valued data and variable to real-valued data and variable.

Q3. Consider the problem, with variable $x \in \mathbf{R}^n$,

$$\begin{aligned} \min_{x \in \mathbf{R}^n} \quad & c^T x \\ \text{s.t.} \quad & Ax \preceq b \text{ for all } A \in \mathcal{A}, \end{aligned}$$

where $\mathcal{A} \subset \mathbf{R}^{m \times n}$ is the set

$$\mathcal{A} = \{A \in \mathbf{R}^{m \times n} \mid \bar{A}_{ij} - V_{ij} \le A_{ij} \le \bar{A}_{ij} + V_{ij}, i = 1, \ldots, m, j = 1, \ldots, n\}.$$

(The matrices $\bar{A}$ and V are given.) Express this problem as an LP.