ELEG5481 Signal Processing Optimization Techniques
Tutorial 6

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Q1. Consider the following problem.

$$\begin{aligned} \min_{x_1,x_2} \quad & f(x_1,x_2) = x_1 + x_2 \\ \text{s.t.} \quad & 2x_1 + x_2 \geq 1 \\ & x_1 + 3x_2 \geq 1 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

(a) Sketch the feasible set and find the optimal solution.
(b) Use the optimality condition to verify the obtained solution is optimal.

Q2. Consider the following three problems, where $a_i \in \mathbf{R}^n, i = 1, \ldots, m$, $b \in \mathbf{R}^m$, and $M > 0$ are given data.

1. The robust least-squares problem

$$\min_x \quad \sum_{i=1}^{m} \phi(a_i^T x - b_i),$$

where $x \in \mathbf{R}^n$, and $\phi : \mathbf{R} \to \mathbf{R}$ is defined as

$$\phi(u) = \begin{cases} u^2, & |u| \leq M, \\ M(2|u| - M), & |u| > M. \end{cases}$$

2. The least-squares problem with variable weights

$$\begin{aligned} \min_{x,w} \quad & \sum_{i=1}^{m} f(a_i^T x - b_i, w_i) \\ \text{s.t.} \quad & w \geq 0, \end{aligned}$$

where $x \in \mathbf{R}^n$, and $f : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ is defined as

$$f(u, w) = u^2/(w + 1) + M^2 w.$$

3. The quadratic program

$$\min_x \quad \sum_{i=1}^{m} g(a_i^T x - b)$$

where $g(u)$ is defined as

$$\begin{aligned} g(u) = \min_{s,t} \quad & s^2 + 2Mt \\ \text{s.t.} \quad & |u| \leq s + t, \\ & 0 \leq s \leq M, \\ & t \geq 0. \end{aligned}$$

Show the following results.
(a) Show that problem (1) and problem (2) are equivalent by showing that

$$\phi(u) = \min_{w \geq 0} f(u, w).$$

(b) Show that problem (1) and problem (3) are equivalent by showing that

$$\phi(u) = g(u).$$