ELEG5481 Signal Processing Optimization Techniques
Tutorial 2

Feb. 3, 2013

Q1. Let $A = U\Sigma V^H$ be a SVD of matrix $A \in \mathbf{C}^{n\times n}$. Show that the problem $\max\{\Re\{\mathrm{tr} AW\} \mid W \in \mathbf{C}^{n\times n}$ is unitary$\}$ has the solution $W = VU^H$, and the value of the maximum is $\sum_{i=1}^{n} \sigma_i$.

Q2. If A is Hermitian, show that there exists a solution $x^\star$ that is optimal to the following two the optimization problems
(a) $v_1 = \max_{x^H x=1} f_1(x) = x^H Ax$.
(b) $v_2 = \max_{x\neq 0} f_2(x) = \frac{x^H Ax}{x^H x}$.

Q3. Prove that for $p \geq 2$,

$$\|x\|_2 \leq n^{-\frac{2-p}{2p}} \|x\|_p,$$

by showing that
(a) The inequality above is true if the following equation is true

$$n^{\frac{2-p}{2}} = \min_{\substack{\|x\|_2=1 \\ x_j\geq 0, j=1,\ldots,n}} \|x\|_p^p. \tag{1}$$

(b) Show that (1) is true for $n = 2$.
(c) Show that (1) is true by induction, i.e. assuming that (1) is true for $n = i - 1$, show that (1) is true for $n = i$.